LEARNING STYLES IN MATHEMATICS – A QUANTITATIVE RESEARCH ON 10TH GRADE PORTUGUESE STUDENTS

ESTILOS DE APRENDIZAJE EN MATEMÁTICAS – INVESTIGACIÓN CUANTITATIVA EN ALUMNOS PORTUGUESES DE GRADO 10

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Abstract
The article reports a research project aimed to describe the learning styles in mathematics of the Portuguese students at the 10th grade, and to analyze how those styles are related to the students' performance. For this quantitative, descriptive and correlational research, I used a questionnaire based on the *ILS* (*Inventory of Learning Styles*) of Vermunt (1994), which I've adapted to the context of the learning of mathematics by the students at the Portuguese secondary schools.
The sample, resulting from a multi-stage method, was composed by 579 students of 28 public schools. Within the conclusions of the research, I emphasize the detection of a learning style that is strongly correlated to the motivational learning orientations and to the self-regulated learning, being however still undefined it what concerns the cognitive processing strategies. This stile, if hold in a favorable context, may turn into a "meaning oriented" learning style. I also detected the four styles usually reported at Vermunt's *ILS'* applications: "meaning oriented", "reproduction oriented", "application oriented" and "not oriented", as well as the positive contribution of a "meaning oriented" style to the scholar performance in mathematics, opposite to the effect of the "reproduction oriented" one.

Keywords: learning styles; mathematics; secondary school

Introduction

The aim of the research was to characterize the learning styles and each of its components in relation to the learning of mathematics by 10th grade Portuguese students and to find out whether either the learning styles or any of their components show any correlation to the learning assessment. Such components are those considered in the Vermunt's model of the regulation of the learning processes (Vermunt and Van Rijswijk, 1988; Vermunt, 1998, 2005): cognitive processing strategies, regulation strategies, conceptions of learning and learning orientations.

1. Theoretical Framework

One of the most used definitions of "learning style" was written by a *task-force* of *NASSP (National Association of Secondary School Principals)*, created in 1979 to set the diagnostics of the learning styles of secondary school students in USA:
"The learning style is the composite of characteristic cognitive, affective, and physiological factors that serve as relatively stable indicators of how a learner perceives, interacts with, and responds to the learning environment. (…)" (Keefe, 2001, p.140)

Sometimes it's not easy to distinguish the concepts of cognitive styles and learning styles (Desmedt & Valcke, 2004). However, most of the cognitive styles theories were developed in laboratorial environments, aiming to find out the individual differences of cognitive processing and revealing a strong correlation between the styles and the personality, whereas the concepts associated to the learning styles came out from the research in educational contexts, in order to explain different ways of learning. The great advantage of the learning styles is that these can be modified by the interaction between the student, the activity and the context (Zhang & Sternberg, 2005).

There is also the concept of approach to learning, which seems to be halfway between the other two above mentioned concepts. It was initially developed by Warton and Säljö in 1976, mentioning three approaches: deep, surface and strategic. These approaches depend on two dimensions of learning: cognitive processing and motivational learning orientations. This two-dimensional concept was a base for the learning styles models developed by Entwistle and Vermunt (Evans & Cools, 2009). Vermunt added two other dimensions, the regulation strategies and the learning concepts of the students, leading to the model that I used in this research.

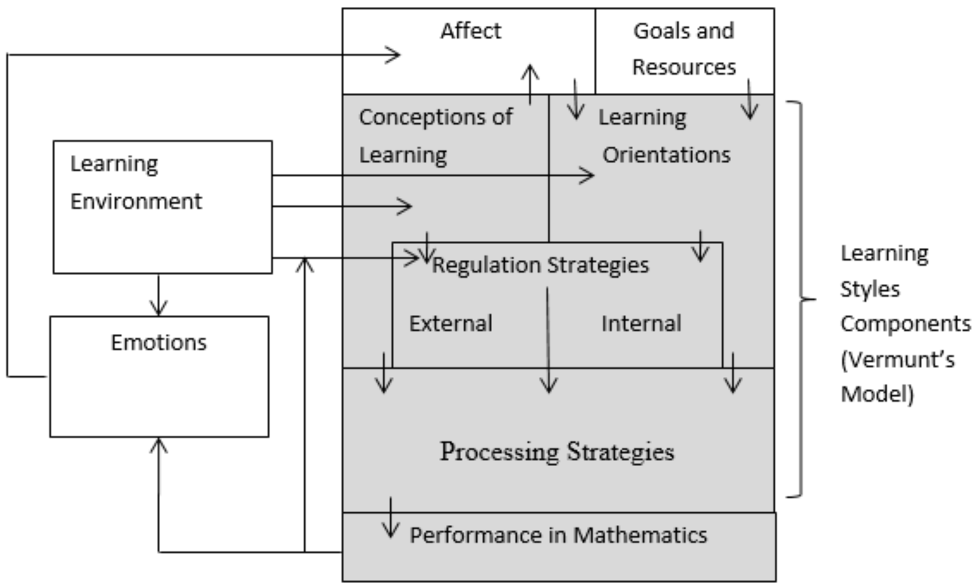

Figure 1. Research conceptual framework (where the variables to be measured are those of the grey area)

The learning styles, in a socio-constructivist insight (Goldin, 1989) that I acknowledge as paradigmatic, are dynamic and their evolution depends on both personal and contextual factors. Though this study is not a longitudinal one, therefore limited by a synchronic data collecting, I conceived a conceptual framework (figure 1) that has got in account some variables that, regardless the fact that they are not targeted to be measured in this research, we assume as being closely related to the four components of Vermunt's model of the regulation of the learning processes. Therefore, the conceptual framework includes this model as a subset that interacts with other variables. For instance, the performance, as perceived by the student, feedbacks the components of the regulation model (Cassidy,

2011) and generates emotions that influence the affect for mathematics (McLeod, 1992). As a result, some changes in the motivation to learn may happen (Hannula, 2004). Besides that, the effect of the perceived performance on the self-confidence of the student may change the degree of the self-regulated learning (Malmivuori, 2006). The learning contexts, such as the school culture and the classroom dynamics, or even the social and familiar environments, are also considered at the framework. The goals of the student and the available resources also have an impact on the learning orientations (Hannula, 2006).

The constructivist model (Vermunt, 1998) defines the learning styles in terms of four learning components: cognitive processing strategies, regulation strategies, conceptions of learning and learning orientations. The different mixes of attitudes and behaviors of a student within each of those components define the learning styles proposed by the model: reproduction oriented, meaning oriented, application oriented and not-oriented.

The Inventory of Learning Styles (ILS) proposed by Vermunt (1994) is composed by the scales: deep, stepwise and concrete processing (three scales for the processing strategies); internal, external and lack of regulation (three scales for the regulation strategies); personally interested, certificate directed, self-test directed, vocation directed and ambivalent (five scales for the learning orientations); intake of knowledge, construction of knowledge, use of knowledge, stimulating education and cooperation (five scales for the beliefs about learning). Some of the scales for the processing strategies and for the regulation strategies are still divided in subscales.

Since this research was targeting secondary school students, I also analyzed the ILS-VE (Vermunt, Bouhuijs & Picarelli, 2003), a version for use at secondary education. The main difference from the former version of Vermunt's ILS was that the new version was provided with a new scale for the emotional aspects of learning, as an answer to some critics that pointed out that in the former version there was few emphasis of the emotional processes induced by the learning context (Coffield et al., 2004).

2. Purpose of the Research

The concept of learning styles is mainly rooted on the disciplinary domain of Psychology. However, when in use together with what Antoli (2008) describes as an object-discipline, such as Mathematics, the concept may be considered as belonging also to the domain of the specific didactics involved. Therefore, with this research, I want to contribute to the knowledge of Mathematics Didactics, providing information about the way that the secondary school students learn mathematics. Nowadays, almost every pedagogical projects refer the importance of having in account the individual differences of the students. Since there are very few empirical studies in Portugal related to this subject, this research looks forward to contribute to the changings of the teaching and learning of mathematics.

3. Objectives

The objectives of this research are to describe the components of the different learning styles of a sample of Portuguese 10th grade students and to find correlations between the learning styles and the performance of those students in mathematics. Therefore, the questions of the study are:

- Which beliefs about the learning of mathematics are predominant among the Portuguese 10th grade students?
- What are the motivational orientations towards the study of mathematics among the Portuguese 10th grade students?
- How do Portuguese 10th grade students regulate their learning?
- Which are the cognitive processing strategies developed by Portuguese 10th grade students at mathematics learning?
- Which leaning styles are more present among Portuguese 10th grade students, at mathematics learning?
- Which correlations can be found between the performance in mathematics and the learning styles or between that performance and each of the four components of the learning styles revealed by Portuguese 10th grade students?

4. Research Method

Having in mind the aim of this research and namely the need of studying the relationships between the learning styles and each component of Vermunt's model, I opted for a quantitative, descriptive and correlational research.

For this study, I targeted the 10th grade students that were learning the discipline "Mathematics A". The advantage of choosing the 10th grade classes is the opportunity to observe students that have just done the qualitative step of the transition from basic to secondary school and that are supposed to be able to interpret the questions written on the inventory.

This large-scale study was preceded by a small-scale study that allowed me to tune up the survey tools. At both studies, I used a multi-stage sampling process, as shown in figure 2. The first stage is a stratified random sampling, through the segmentation of the population in strata that are mutually exclusive and exhaustive, each one subjected to a simple random sampling. The second stage is a clusters selection, each one corresponding to a class of students. The selection followed the convenience of both researcher and school. This is not a randomized process but, assuming that there is no correlation between the criteria of convenience used by both parts, researcher and school, the effect can be seen as similar to the effect of a random process. Once a class was selected, all the pupils in the classroom fulfilled the inventory. I must emphasize that the criteria of the class selection that I used as researcher was the will of being present at the data collection, assuring the homogeneity of the process within all the selected schools, whilst the school criteria concerned the compliance with the curricular plan and scheduling.

Concerning the size of the sample, this was determined by a maximum sample deviation of one tenth of each unit of the measuring scale used at the wide-range research and a level of confidence of ninety-five percent. For this research, I also decided that all the administrative regions of continental Portugal had to be represented in the sample by at least one school. For the wide-range study, the calculation of the size of the sample pointed to a minimum of 417 students. Assuming a minimum average number of present students per class equal to 15, a conservative number per precaution, I came to the conclusion that at least 28 classes should be surveyed.

The 28 classes selected through the second stage, one per school, were distributed by the regions as follows: 11 classes in the North, 8 classes in Lisbon and Tagus Valley, 6 classes in the Center, 2 classes in Alentejo and 1 class in Algarve, totalizing a sample of 579 students.

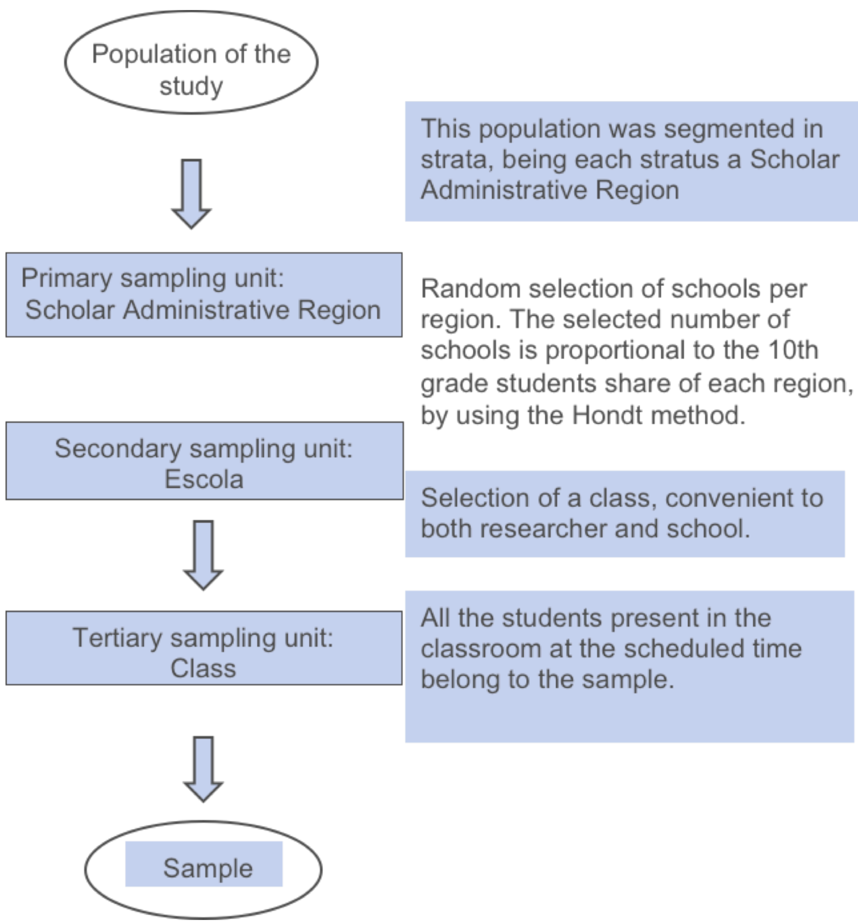

Figure 2. Sampling method

The tool used for the primary data collection was an adaptation of Vermunt's ILS – Inventory of Learning Styles (Vermunt, 1994). I adapted this questionnaire to a secondary school context and to the focus on the learning of mathematics. In this adaptation, I kept the structure of the Likert scales and subscales used at the original inventory for measuring the attitudes and behaviours related to the four components of Vermunt's model (see table 1). Each scale or subscale is composed by five questions with four alternative answering positions. As a whole, the adapted ILS comprises one hundred closed questions related to the components of the learning styles. The questionnaire also included three questions related to the results in Mathematics (self-assessment, classifications in the former year and in the current year) and two questions about the demographic variables of gender and age.

Table 1. *Scales and subscales of ILS' components*

Component	**Scale**	**Subscale**
Processing Strategies	Deep Processing	Relating and Structuring
		Critical Processing
	Stepwise Processing	Memorizing and Rehearsing
		Analysing
	Concrete Processing	
Regulation Strategies	Internal Regulation	Regulation of Learning Processes and Results
		Regulation of Learning Contents
	External regulation	Regulation of Learning Processes
		Regulation of Learning Results
	Lack of Regulation	
Learning Orientations	Personal Interested	
	Certificate Directed	
	Self-test Directed	

	Vocation Directed	
	Ambivalent	
Beliefs about Learning	Intake of Knowledge	
	Construction of Knowledge	
	Use of Knowledge	
	Stimulated Education	
	Cooperation	

For the two first components, that concern the processing and the regulation strategies, the scales are behavioural. For example:

"When a mathematical problem is presented at the classroom, I prefer to wait for an explanation about how to solve it, either from the teacher or from my colleagues"

Never o Sometimes o Often o Always o "

For the other two components, the scales are attitudinal. For example:

"I like to learn Mathematics

I totally disagreeo I tend to disagreeo I tend to agreeo I totally agreeo"

The field work for collecting the data was done in the year 2016, between 15th January and 15th March at 28 schools. The researcher was always present at the classroom during the fulfilling of the questionnaire by the students, in order to assure the homogeneity of the procedures and to guarantee the anonymity of the answers given by the students.

The data analysis was performed by using IBM SPSS™, a statistical software package for social sciences. Firstly, this was used for the internal validation of the questionnaire, through the calculation of the kurtosis and the skewness of the variables distribution and also by determining the Cronbach's alpha of the scales. Afterwards, some descriptive statistics were used for characterizing the composition of the sample and to analyse the components of the learning styles. These were found out by using a factorial analysis of principal components, after having tested the adequacy of the sample for such analysis. Tests for comparing means and variances were also performed for a better description of the attitudes and behaviours related to the components of the learning styles. In order to answer the last question of the research, about finding relationships between the learning styles or their components and the mathematics performance, I used Pearson's correlation factor tests.

5. Findings

The findings here reported are those of the large-scale study. However, it is remarkable that these findings were very similar to the findings of the pilot-study, since that can be seen as a confirmation of the validity of the questionnaire used for data collection.

The sample was quite balanced in what concerns the gender of the students: 52% female and 48% male. Most of them (78%) were 15 years old and there was a significant group (18%) aged 16 years. The minimum age was 14 and the maximum age was 18. Concerning their performance in Mathematics, only 16% of the surveyed students assess their performance as non-satisfactory. However, both school assessments of the 9th year and of the first trimester of the 10th year point out to 22% non-satisfactory performances.

The statistical description of the processing strategies used by the students is summarized in Table 2 and shows that the mean for the scale about deep processing scored higher than those of stepwise processing and concrete processing. The equality of the mean scores of these two scales is not rejectable, according to the submission of the results to a t-test. The correlation between the processing strategies and the performance in Mathematics, shows to be positive and significant for the deep processing, and significantly negative for the two other strategies, regardless the scale used for the assessment.

Table 2. *Statistical description of the processing strategies*

Scale	Alpha (Cronbach)	Mean	Variance	Maximum	Minimum
Deep	0,720	2,812	0,135	3,524	2,395

processing					
Stepwise processing	0,686	2,282	0,184	3,010	1,794
Concrete processing	0,627	2,257	0,107	2,736	1,856

Concerning the regulation strategies, Table 3 shows that the highest mean concerns the Internal Regulation. Since the scales used range from 1 to 4, this value is slightly above the average of the scale (2,5), as it happens to the processing strategies. The correlation between the regulation strategies and the performance in Mathematics, shows to be positive and significant for the internal regulation, and significantly negative for the two other strategies, regardless the scale used for the assessment. I also found out that the internal regulation is strongly correlated to the deep processing, whereas the other regulation strategies show significant correlations to the stepwise processing.

Table 3 - Statistical description of the regulation strategies

Scale	Alpha (Cronbach)	Mean	Variance	Maximum	Minimum
Internal regulation	0,749	2,798	0,328	3,467	1,956
External regulation	0,705	2,157	0,113	2,779	1,747
Lack of regulation	0,748	2,085	0,070	2,541	1,858

The highest mean score for the learning orientation scales (see Table 4) refer to the self-test motivation. Only the ambivalence of the learning orientations scored under the average of the measuring scale. The hypothesis of equality of means between the personally interested orientation and the vocational orientation can't be rejected.

Table 4. *Statistical description of the learning orientations*

Scale	Alpha (Cronbach)	Mean	Variance	Maximum	Minimum
Personally interested	0,838	2,945	0,072	3,208	2,682
Certification oriented	0,627	2,704	0,266	3,226	2,194
Self-test oriented	0,681	3,191	0,015	3,268	3,050
Vocation oriented	0,895	2,999	0,044	3,237	2,686
Ambivalent	0,763	2,018	0,136	2,373	1,554

All the correlations calculated between the learning orientations and the performance in mathematics are significant, but weak. The personal interested, the vocational and the self-test orientations correlate positively with all the performance assessments in use, opposite to what happens to the other learning orientations. This might be expected, because those orientations require somehow a positive attitude towards mathematics.

The first remarkable observation in what concerns the beliefs about learning is that, in general, the measuring scales for this component of the learning styles had lower Cronbach's alpha than the other scales. This may happen due to the fact that the concepts involved in the questions about those beliefs are probably not yet clear at the students mind. In fact, three of the five scales revealed a Cronbach's alpha very close to the minimum acceptable value of 0,5, as shown in Table 5.

Table 5. *Statistical description of the learning orientations*

Scale	Alpha (Cronbach)	Mean	Variance	Maximum	Minimum
Intake of knowledge	0,572	2,655	0,072	2,997	2,354

Construction of knowledge	0,511	3,239	0,013	3,371	3,156
Use of knowledge	0,509	3,089	0,020	3,296	2,986
Stimulated Education	0,641	3,322	0,086	3,671	2,889
Cooperation	0,757	2,766	0,154	3,263	2,241

Though the performed t-tests show that we can statistically accept that the means of these scales are all different, they fit above the average of the measuring scale and range from 2,6 (learning seen as an intake of knowledge) to 3,3 (learning seen as a result of educational stimula). Since this results show a mix of beliefs shared by many students, it is expected that there are no strong correlations between this beliefs and the performance in mathematics. In fact, the only significant, however low, correlations that were found are those concerning the beliefs in the intake of knowledge and cooperation, both correlating negatively with performance in mathematics, and the one that refers to the construction of knowledge, with a positive correlation to the assessments used in this study. If there is a trend for those who are less succeeded in mathematics to believe in cooperation, it is possible that, in the beginning of the secondary studies, many students have a view of group work as an opportunity to be positively accessed, rather than an opportunity to learn better.

The results above suggest that there are scales of different components of the learning styles that are correlated, so a factorial analysis may find out common trends of attitudes and behaviours at mathematics learning. As expected, the Kaiser-Meyer-Olkin test value reached nearly 0,9 and the significance level obtained at the Bartlett test was less than 0,01, so both tests revealed that the sample was very good for this purpose. Proceeding to the factorial analysis and selecting the principal components with eigenvalues higher than 1, I could obtain 5 components that explain 70% of the variance within the sample (Table 6).

Table 6. *Extraction of principal components through factorial analysis*

Component	Eigenvalue	Variance (%)	Accumulated Variance (%)
1	3,697	23,108	23,108
2	2,382	14,889	37,997
3	2,120	13,250	51,247
4	1,901	11,881	63,128
5	1,066	6,663	69,790

The factorial structure of these components in terms of the ILS' scales is shown by the matrix of Table 7, where we can find out the correlational saturations after the use of Varimax rotation at the analysis.

Table 7. *Factorial structure with 5 components*

	Component				
Scale	1	2	3	4	5
Deep processing			,828		
Stepwise processing		,856			
Concrete processing				,637	
Internal regulation	,408		,775		
External regulation		,604	-,426		
Lack of regulation	-,396	,445	-,429	,355	
Personally interested	,870				
Certification oriented	-,544	,361			
Self-test oriented	,730				

Vocation oriented	,838				
Ambivalent	-,725		-,364		
Learning as intake of knowledge		,838			
Learning as construction of knowledge			,338	,625	
Learning as use of knowledge	,564			,440	
Learning as stimulated education				,738	
Learning through cooperation					,971

The first component is strongly associated to the three learning orientations that result from the motivation to learn mathematics: personal interest, self-test and vocational orientations. So, it can represent a learning style that has got features that belong to the "meaning oriented" style, but it is not correlated neither to the deep processing strategies, nor to the belief that learning is knowledge construction. I named this style "personal fulfilment oriented" and I propose the conjecture that this style may evolve to the "meaning oriented" one, if the context of learning is favourable to this evolution. This is just a conjecture that must be submitted to longitudinal research. The second component shows features that correspond to the "reproduction oriented" style of Vermunt's model, namely the stepwise processing, the external regulation, the certification orientated learning and the belief that learning is an intake of knowledge. The third component sticks totally to the attitudes and behaviours associated to the "meaning oriented" learning style, such as the deep processing, the internal regulation and the belief of learning as a construction of knowledge. What is noticeable is that the motivational aspects are very strong at the "personal fulfilment oriented" style, but almost absent in the "meaning oriented" one. The interpretation of this fact is that the first one, observable at the beginning of secondary school is more determined by the discipline-object than the latest, which is not so dependent on motivation, thus more stable. The fourth component that was extracted at the factorial analysis reveals a style similar to the “application oriented” one of Vermunt’s model, though not so clearly defined as the other styles. The main features of this style are the relevance of the concrete processing and the belief that learning is a result of educational stimulation. The fifth component is absolutely undefined in what concerns the processing and regulation strategies as well as the learning orientations, so it may be seen as a “not oriented” style. However, in this study, this style appears strongly and exclusively related to the belief in learning through cooperation. Having in mind that, as mentioned above, the correlation of this belief with the performance in mathematics tends to be negative, it is possible that the individual lack of learning strategies at this school level leads some students to look for support at group work. It is noticeable that, comparing this results to those of the pilot-study of the research, the obtained learning styles are the same, however better defined by the large-scale study. Even this special result observed at the “not-oriented” style appeared already at the outcomes of the pilot-study.

Observing the results that concern the linear correlation between the learning styles and the performance in mathematics assessed through three different ways (Table 8), we find levels of correlation that are low, but only one is not significant for p<0,5.

Table 8. *Correlation between the learning styles and the performance in mathematics*

Learning Style		Self-assessment	School assessment (9th grade)	School assessment (10th grade, 1st trimester)
Personal fulfilment oriented	Pearson’s r	,453	,284	,328
	Sig. (bilateral)	,000	,000	,000
	N	578	579	574
Reproduction oriented	Pearson’s r	-,233	-,256	-,346

	Sig. (bilateral)	,000	,000	,000
	N	578	579	574
Meaning oriented	Pearson's r	,281	,181	,248
	Sig. (bilateral)	,000	,000	,000
	N	578	579	574
Application oriented	Pearson's r	-,091	-,027	-,115
	Sig. (bilateral)	,029	,517	,006
	N	578	579	574
Not oriented	Pearson's r	-,152	-,144	-,148
	Sig. (bilateral)	,000	,000	,000
	N	578	579	574

It is however important to notice that the polarity of the correlations is coherent along the three different assessments. The correlations with the assessments are positive for the "personal fulfilment oriented style" and for the "meaning oriented style" and negative for all others. So, I conclude that there is a slight but observable trend of these learning styles to generate a better performance in mathematics, whereas the other styles conduct to worse results. It's also noticeable that the positive correlations of the styles with the assessments are more positive with the self-assessment than with the other assessments and the negative correlations are more negative with the school assessments than with the self-assessment. The interpretation is that those whose learning styles are predominantly "personal fulfilment oriented" or "meaning oriented" are more aware of the results of their learning processes. This interpretation is coherent with the observation that the internal regulation processes only can be assigned to this two styles, according to the factorial structure shown in Table 7.

6. Conclusions

Starting to discuss the results by the constitutive components of the learning styles as defined by Vermunt's model of learning processes regulation, I conclude that, in what concerns the beliefs about mathematics learning, the students at the 10th grade didn't develop clear concepts. Having in mind that most of them are 15 or 16 years old, it wouldn't be expectable that, at these ages, the cognitive processes related to the construction of such kind of concepts would be enough developed. Besides that, it was noticeable that there is no use to discuss these beliefs in the classroom. Some students asked their teachers to discuss some items of the research questionnaire, immediately after having completed it. I believe that, along the secondary studies, the students can define better their conceptions about learning. For this purpose, the affective structure created by the learning contexts plays a major influence on the beliefs of the students (Gomez-Chácón, 2000), so it is very important to create a supportive emotional ambience for the students (Hannula, 2004). Though the results of the research don't show outstanding beliefs about mathematics learning, there are some slight trends to perceive this learning in terms of the educational stimulation and also as a construction and use of knowledge. The empirical studies that I found on this subject in secondary schools were conducted in the Netherlands by Severiens and Dam (1997) and by Könings, Brand-Gruwel and Elen (2012). The results of both studies point out to the same trends for learning beliefs.

Observing the results about the learning orientations, I emphasize that in the investigation, as well as at Severiens and Dam (1997)'s research, the motivational dimensions are those that play the strongest differentiating role between the learning styles in mathematics of secondary school students. In both researches, a learning style different than those found at other applications of Vermunt's ILS comes out, due to preponderance of the motivational factors associated to the learning orientations, so it may be a specific stile of early secondary school students. Since in the study there was an immediately previous choice of a course curriculum that involved Mathematics as a crucial discipline, this result makes sense. From these results, I believe that it is very important that all the contextual factors, such as the teaching methods and styles, keep this motivational predisposition in a high level, in order to give way to the evolution towards a meaning oriented style. If too much stress is put on the assessment of mathematics performance, especially when such assessment is required for any kind of certification, it may cause a drift to a certification oriented learning and therefore to a learning style of a more reproductive kind, which tends to lower the performance of the student in mathematics.

As expected from the results of the learning orientations, the self-regulated learning is more present at this sample than the external regulation. However, this trend is still soft when the learning style is "personal fulfilment oriented", but much higher when the "meaning oriented" learning style overcomes. The results of this research are aligned with the self-regulation learning model proposed by Boekaerts (1999), placing the affective attitude towards learning as the first step towards the self-regulation, and they also are congruent with the results obtained by Hannula (2004) and Malmivuori (2006) about the relations between affect, motivation and self-regulation skills.

The pattern of the cognitive processing strategies is similar to the one about the regulating strategies, since there is a kind of strategy that is more relevant, the deep processing one, but its relevance, though significant when comparing to the other strategies, is moderate. This result is aligned with what I've observed at the other components of the learning styles. As reported by other studies, the deep processing strategies are strongly correlated with the vocational orientation (Duarte, 2007) and with the self-regulated learning (Vermunt, 1998). A question that arises from these results is whether the deep processing strategies remain relevant along the whole secondary school cycle. In a study about cognitive processing strategies of secondary school Portuguese students, Gomes (2006) observed that the use of the deep processing strategies rise from the 10^{th} to the 11^{th} grade, but go lower by the 12^{th} grade students. Two contextual factors are hypothetical explanations for this observation: the need of high results in exams that filter the access to higher studies and the pressure of the school rankings over teachers and school administrations, both driving to certificate oriented learning and to reproduction oriented processes. It was also observable in a longitudinal research in the Netherlands by Könings, Brand-Gruwel and Elen (2012), that in the last year of the secondary school studies, there is less use of deep processing cognitive strategies by the students, along with the lowering of their expectations.

In the research I could detect the four learning styles that were reported in other studies (Severiens & Dam, 1997; Vermetten, Vermunt & Lodewijks, 1999; Boyle et al, 2003; De Meyer & Van Petegem, 2003; Vermunt, 2005; Rocha & Ventura, 2011), and also another learning style that seems to appear only at secondary school learning. Those five styles can be described as follows, by order of relevance observed at the sample:

- "Personal fulfilment oriented" – the main features of this style are the motivational orientations related to the personal interest, to the self-test and to the individual vocation of the students. This style can be also defined by a trend to self-regulated learning and to the view of learning as a process of knowledge use. However there are no cognitive processing strategies clearly assigned to this style. The conjecture about this style is that this is a kind of proto-style, present at least at the first year of secondary school, and able to develop to a meaning oriented style, if the contextual factors are suitable to induce a concept of learning as construction of knowledge and a trend to wider use of deep processing strategies.
- "Reproduction oriented" – As observed in other studies, the stepwise processing strategies, the external regulation and the view of learning as knowledge intake are the key factors of this style. Another factor, still significant, is the learning orientation towards certification.
- "Meaning oriented" – This style is strongly characterized by the deep processing strategies and by the self-regulated learning and also, though not so intensively, by the concept of learning as knowledge construction.
- "Application oriented" – The main aspects of this style are the concrete processing strategies and the beliefs that learning is dependent on the educational stimuli, but also, in a lower degree, it is a process of both construction and use of knowledge. Another feature of this style is that there is few regulation of learning.

- “not oriented” – This style is completely undefinable in terms of all the components of the learning styles, except what concerns the concepts of learning. There is an exclusive and very strong assignment of this style to the view of learning as a cooperative process.

Answering the last question of the research, I found out some relations, significant but not strong, between the learning styles and the mathematic performance of the students, measured through three different indicators. I concluded that the results of mathematic learning tend to be more positive by the use of deep processing strategies, by self-regulated learning, by the concept of learning as construction of knowledge and by motivational factors connected to a positive affection for mathematics learning. In coherence with the above description of the learning styles, I also concluded that the “personal fulfilment oriented” and the “meaning oriented” styles tend to contribute to a better performance in mathematics. This performance tends to be negatively affected by the styles of learning that are “reproduction oriented” or “not oriented”, and by the strategies, beliefs and learning orientations that are preponderant at these styles. I could also observe that those who revealed “personal fulfilment oriented” and “meaning oriented” styles are more aware of their learning results. This may be explained by the preponderance of self-regulating learning in both styles.

Having exposed the conclusions of the research, I must express some limitations that are inherent of the methodology. First of all, one may ask whether this results can be generalized to the population of the study. There are some factors in the sampling process that could cause interference in the randomness of sample: the convenience of the selection process of the classes, the risk of having selected classes instead of individual students, considering that some of these might be absent at the moment of the data collection, and the stratification of the population by regions in order to obtain a representative territorial distribution of the sample. If we define a random process in terms of the equiprobability of selection for each sampling unit, the assumed no correlation between the conveniences of the researcher and the schools makes this part of the sampling equivalent to a random process. Concerning the absent students, generally there were no more than one or two missing the class. Therefore, I consider that there is a high degree of generalization of the research results and that another sampling process would hardly obtain a so close approach to a random process. For such kind of investigations, there are practical limitations that avoid a pure random sample. There are also limitations that result from the data collection process. The answers given by the students can be influenced by subjective norms, in the sense that they may consider answers in terms of the opinion of contextual agents and not of their own. Besides that, some answers may require cognitive constructs that need the use of the memory of the thoughts and emotions that have occurred at mathematics learning. The possible misinterpretation of some questions and terms might have been a limitation too. However, I believe that these limitations had few impact at the results, since the results of the pilot study and of the large scale study were very similar.

Some follow up of this research would contribute to further knowledge about the learning styles in mathematics of the secondary school students. Longitudinal studies along the secondary school cycle would help to describe the evolution of the learning styles and to test the conjecture that there is a learning-style that can turn into a meaning oriented style, if some contextual conditions are favourable to this development. These studies could also check to which extent the assessment methods at the end of the secondary school are influencing the learning styles adopted by the students. Another interesting question, that arises from the results of this study and may induce further research, concerns the understanding by secondary school students of what cooperative learning means. Those who are not oriented at their learning were strongly associated to this view of the learning process. Since the results also point out to a significant, though low, negative correlation between this learning concept and the performance in mathematics, there’s the challenge to find out whether the students assign this concept to a way of obtaining better classifications through the collective assessment of the group, rather than an effective mathematics learning that might result from the team work. It would also be interesting to try to replicate this results of this research in studies focusing other mathematical curricula, namely those who are taught to vocational secondary school students.

The main contributions of this research to the scientific domain of Mathematic Didactics are those from the methodology, namely the sampling method and the adaptation of Vermunt's ILS, and those of the conclusions that can be used as a basis for further investigations and also may give some cues for the teachers work.

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